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LETTER TO THE EDITOR

A renormalisation group approach for two-dimensional site percolating system

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Abstract. A renormalisation group approach is applied to the square site percolating system using a scaling transformation of a finite lattice in real space. The scale of length b for renomalisation has been changed for $b=2, 3$ and 4. For each scale of length, the location of the fixed point and the correlation length critical exponent are calculated and listed in tabular form.

In this Letter we present a renormalisation group approach for calculating the critical concentration p_c and the correlation length critical exponent ν of the two-dimensional site percolating system. The method can be set up in terms of Kadanoff's original picture, the block spin picture (Kadanoff 1966), using the renormalisation group theory. The method in Kadanoff's picture has been formulated recently to study critical behaviour in percolating systems (Harris *et al* 1975, Young and Stinchcombe 1975, Stinchcombe and Watson 1976). If the probability p of a site with non-zero conductance on the original lattice scales into a new probability p' of a site on the new lattice by the renormalisation transformation R:

$$p' = R(p), \tag{1}$$

then the fixed point p^* is determined by

$$p^* = R(p^*). \tag{2}$$

The non-trivial fixed point gives the approximation for the critical percolation probability p_c. The linearised form R^L of the renormalisation transformation near the fixed point has eigenvalues λ_i with $\lambda_1 > 1 > \lambda_2 \ldots$. The correlation length critical index ν is given by

$$b^{1/\nu} = \lambda_1, \tag{3}$$

where b is the change of scale of length.

The basic scaling procedure is defined by a simple renormalisation transformation on finite lattices. In figure 1(a), an illustration of the basic scaling procedure is provided by which the original square lattice (full line) scales into the new lattice (broken line) with a scale factor $b=2$: the original cluster group of the sites of the 2×2 lattice (which we call cell) in figure 1(a) scales into a single site with probability p'. As shown in figure 1(b) the renormalised probability p'. As shown in figure 1(b) the

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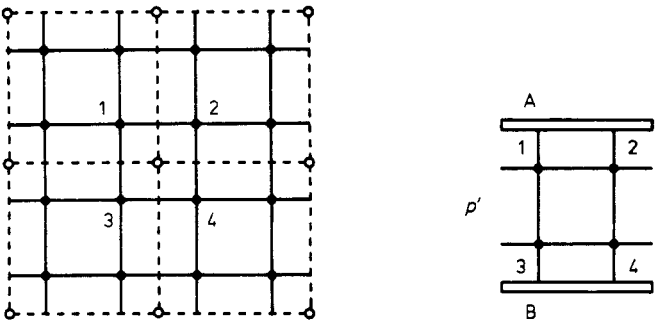

Figure 1. (*a*) Transformation of the square site lattice: the full line is an original lattice; the broken line is the transformed lattice with the scale factor $b = 2$. (*b*) Combination of paths for the new probability p'. Cluster group of sites 1–4 sandwiched between two plane electrodes A and B and two sides with cyclic boundary conditions.

renormalised probability p' is determined as the probability of the cell being conductive when the cell is sandwiched between two plane electrodes A and B made of perfect conductor: two sides of the cell are connected to each other with cyclic boundary conditions to eliminate edge effects. We can obtain the transformed probability $R_2(p)$ from the paths which contribute to the conductance of the cell according to the exclusion–inclusion principle:

$$R_2(p) = 2p^2 - p^4. \tag{4}$$

From this scaling transformation (4), the fixed point p_2^*, the eigenvalue λ_1 and the correlation length critical index ν_2 of the 2×2 lattice approximation are given by

$$p_2^* = 0{\cdot}6180, \qquad \lambda_1 = 1{\cdot}528, \qquad \nu_2 = 1{\cdot}635. \tag{5}$$

We also carry out the calculations of the 3×3 and 4×4 cell approximations. Similarly to the 2×2 cell approximation the probability p of the 3×3 and 4×4 site connections on the original lattice scales into the probability p' for the connection between two plane electrodes and the sides with the cyclic boundary condition. The probabilities of a single site on the new lattice for 3×3 and 4×4 lattice approximations are respectively

$$R_3(p) = 3p^3 + 6p^4 - 12p^5 - 3p^6 + 12p^7 - 6p^8 + p^9, \tag{6}$$

$$R_4(p) = 4p^4 + 16p^5 - 64p^7 - 66p^8 + 320p^9 - 284p^{10} + 40p^{11} + 50p^{12} - 8p^{13} - 8p^{14} + p^{16}. \tag{7}$$

It is noted that the expressions (7) and (8) are exact expressions for the percolation probability of the finite 3×3 and 4×4 lattices with cyclic boundary conditions respectively.

Our results for the fixed point p^*, the eigenvalue λ_1 and the correlation length critical exponent ν for each cell approximation are shown in table 1. The fixed point and the critical exponent become smaller as the scale factor b becomes larger. The decrease of the fixed point is nearly linear as a function of b^{-2} within this cell approximation. If we assume the linear dependence of p^* on b^{-2} for large b, we can

Table 1. Results of cell approximations to the renormalisation group for site percolating square lattice.

Size of cells	Fixed point p^*	Eigenvalue λ_1	Exponent† ν
2×2	0·6180	1·528	1·635
3×3	0·6023	1·972	1·618
4×4	0·5945	2·379	1·599

†Correlation length critical exponent.

estimate the value of fixed point for $b\to\infty$ and obtain $p^*=0{\cdot}586$: this value is very near $2-\sqrt{2}$. The critical percolation probability p_c for the site percolating square lattice is known to be $0{\cdot}590\pm0{\cdot}005$ from the Monte Carlo method (Niel 1972) and $0{\cdot}587\pm0{\cdot}005$ from computer simulation on a 500×500 site lattice (Y Yuge and K Onizuka 1977, private communication). Our result for the fixed point $p^*=0{\cdot}5945$ for the 4×4 cell approximation is in good agreement with the value of critical concentration p_c obtained by Niel; our rough estimate value $p^*=0{\cdot}586$ for $b\to\infty$ is in good agreement with the value obtained by Yuge and Onizuka.

The critical exponent ν for the correlation length was obtained by computer simulation (Harris *et al* 1975). The value is $\nu=1{\cdot}5\pm0{\cdot}2$. Our results for the correlation length critical exponent are all consistent with the results of the computer experiment.

From these comparisons of our results with the results of computer experiments we can conclude that our procedure of the renormalisation group approach is a very simple but rather good method for estimating the critical concentration p_c. Further computation by this method will give more accurate estimates for the critical concentration p_c etc; it is however difficult to calculate further because of the long machine time of the computer.

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